

Charged Lepton Flavor Violating Processes and Scalar Leptoquark Decay Branching Ratios in the Colored Zee-Babu Model

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Neutrino mass generation

- $m_\nu \neq 0$. Needs to go beyond the SM
- Key for Majorana ν : Weinberg operator; the only dim-5 SM inv. eff. operator, $L \neq 0, B = 0$.

$$\frac{(LH)^2}{\Lambda}$$

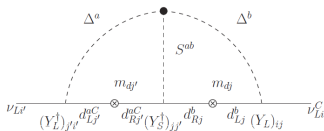
- dilepton $L \neq 0, B = 0$: see-saw 1,2,3.
- Leptoquark (LQ) $L \neq 0, B \neq 0$ is also a legitimate candidate
- To have $B = 0$, either (1) 2 LQs with $L_1 : L_2 \neq B_1 : B_2$ or (2) LQ + Di-quark (DQ) with $L = 0, B \neq 0$.
- We go for 1LQ+1DQ and address the connection between M_ν and experimental signatures.

- LQ mass direct search bound (95% CL., GeV, $\beta = 1(0.5)$). LQ decay BR into lq and νq denoted as β and $(1 - \beta)$, and λ : Yukawa coupling for $lq\Delta$. The LQ is assumed to decay into leptons within only one specific generation.

	1st gen.	2nd gen.	3rd gen.
CMS	1005(845)	1070(785)	634
ATLAS	660(607)	685(594)	534
ZEUS	699($\lambda = 0.3$)		

- For an E_6 -type DQ, CMS study gives $m_S > 6\text{TeV}$.
- Very sensitive to the assumptions of decay BR as well as the flavor dependant coupling strengthes.
- We take $m_S = 7\text{TeV}$ and $m_\Delta = 1\text{TeV}$ as the benchmark values

Colored Zee-Babu Model



- Relevant Lagrangian for $\Delta(3, 1, -1/3)$ and $S(6, 1, -2/3)$:

$$\begin{aligned}
 & - \left[\overline{L_i^C} (Y_L)_{ij} i\sigma_2 Q_j + \overline{(\ell_{Ri})^C} (Y_R)_{ij} u_{Rj} \right] \Delta^* - \overline{(d_{Ri})^C} (Y_S)_{ij} d_{Rj} S^* \\
 & \quad + y_{ij}^\Delta \overline{(u_{Ri})^C} d_{Rj} \Delta + \mu \Delta^* \Delta^* S + h.c.
 \end{aligned}$$

i, j : flavor indices, $SU(3)$ indices suppressed. Y_S : symmetric in flavor

- Proton decay $\propto (Y_{L/R} Y_{11}^\Delta)^2$ can be evaded. (1) A very small y_{11}^Δ . (2) y^Δ can be eliminated by for example some Z_2 parities $\{-, -, +, +, +, -, +\}$ assigned to $\{L, l_R, Q, u_R, d_R, \Delta, S\}$ respectively.

Colored Zee-Babu Model

$$(M_\nu)_{ii'} = 24\mu(Y_L)_{ij}m_{dj}I_{jj'}(Y_s^\dagger)_{jj'}m_{dj'}(Y_L^T)_{j'i'} ,$$

$$I_{jj'} = \int \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} \frac{1}{(k_1^2 - m_{dj}^2)} \frac{1}{(k_1^2 - m_\Delta^2)} \frac{1}{(k_2^2 - m_{dj'}^2)} \frac{1}{(k_2^2 - m_\Delta^2)} \frac{1}{(k_1 + k_2)^2 - m_S^2}$$

$$I_{jj'} \simeq I_\nu \equiv \frac{1}{(4\pi)^4} \frac{1}{M^2} \frac{\pi^2}{3} \bar{I}\left(\frac{m_S^2}{m_\Delta^2}\right), \quad M \equiv \max(m_\Delta, m_S)$$

$$\bar{I}(x) = \begin{cases} 1 + \frac{3}{\pi^2}(\ln^2 x - 1) & \text{for } x \gg 1, \\ 1 & \text{for } x \rightarrow 0. \end{cases}$$

- Write $M_\nu = Y_L \omega Y_L^T$, $\omega_{jj'} \equiv 24\mu I_\nu m_j m_{j'} (Y_s^\dagger)_{jj'}$.
- Qualitatively,

$$m_\nu \sim \frac{\mu m_b^2 Y_L^2 Y_S}{32\pi^2 M^2} \sim 0.06\text{eV} \times \left(\frac{Y_L^2 Y_S}{10^{-6}}\right) \times \left(\frac{\text{TeV}}{M^2/\mu}\right) .$$

2-loop, typical values $Y_L, Y_S \sim 0.01$ and $\mu, M \sim 1$ TeV, sub-eV m_ν w.o. excessively fine tuning.

- scaling: $Y_L^2 Y_S = \text{const}$, trivial but useful.

Iterative solution for Y_L

- Working assumption: Democratic Y_S (easy in extra-dim).
- Since $m_b \gg m_s \gg m_d$

$$\omega^{(0)} = 24\mu I_\nu \times \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & m_b m_s (Y_S)_{23}^* \\ 0 & m_b m_s (Y_S)_{23}^* & m_b^2 (Y_S)_{33}^* \end{pmatrix}$$

- (1) $\mathcal{O}\left(\frac{\omega^{(1)}}{\omega^{(0)}}\right) \sim \mathcal{O}\left(\frac{m_d}{m_b}\right)$. (2) $M_\nu^{(0)} = Y_L \omega^{(0)} Y_L^T$ is of rank-2,
 (3) $\det M_\nu^{(0)} = 0$. (4) Lightest one $\sim (m_d/m_b) \times \max(m_\nu)$,
 (5) quasi-degenerate disfavored.

- once $\{\mu, m_S, m_\Delta, (Y_S)_{13,23,33}, (Y_L)_{13}\}$, Y_L can be determined

$$(Y_L)_{23}^{(0)} = \frac{(Y_L)_{13}}{(M_\nu)_{11}} \left[(M_\nu)_{12} \pm \sqrt{(M_\nu)_{12}^2 - (M_\nu)_{11}(M_\nu)_{22}} \right],$$

$$(Y_L)_{32}^{(0)} = \frac{(Y_L)_{13}}{(M_\nu)_{11}} \left[(M_\nu)_{13} \pm \sqrt{(M_\nu)_{13}^2 - (M_\nu)_{11}(M_\nu)_{33}} \right],$$

$$(Y_L)_{12}^{(0)} = \frac{(M_\nu)_{11} - B_s m_b^2 (Y_L)_{23}^2 (Y_S)_{33}^{(0)}}{2B_s m_b m_s (Y_L)_{13} (Y_S)_{23}^{(0)}},$$

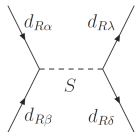
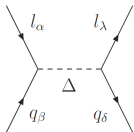
$$(Y_L)_{22}^{(0)} = \frac{(M_\nu)_{22} - B_s m_b^2 (Y_L)_{23}^2 (Y_S)_{33}^{(0)}}{2B_s m_b m_s (Y_L)_{23} (Y_S)_{23}^{(0)}},$$

$$(Y_L)_{32}^{(0)} = \frac{(M_\nu)_{33} - B_s m_b^2 (Y_L)_{33}^2 (Y_S)_{33}^{(0)}}{2B_s m_b m_s (Y_L)_{33} (Y_S)_{23}^{(0)}},$$

$$(Y_L)_{11}^{(1)} = -\frac{m_s}{m_d} \frac{\delta_{23}}{(Y_S)_{13}} (Y_L)_{12}^{(0)} - \frac{m_b}{2m_d} \frac{\delta_{33}}{(Y_S)_{13}} (Y_L)_{13},$$

$$(Y_L)_{21}^{(1)} = -\frac{m_s}{m_d} \frac{\delta_{23}}{(Y_S)_{13}} (Y_L)_{22}^{(0)} - \frac{m_b}{2m_d} \frac{\delta_{33}}{(Y_S)_{13}} (Y_L)_{23}^{(0)},$$

$$(Y_L)_{31}^{(1)} = -\frac{m_s}{m_d} \frac{\delta_{23}}{(Y_S)_{13}} (Y_L)_{32}^{(0)} - \frac{m_b}{2m_d} \frac{\delta_{33}}{(Y_S)_{13}} (Y_L)_{33}^{(0)}.$$



$$\begin{aligned}
 \Delta \mathcal{L}_{\text{eff}} = & \left[\frac{(Y_L^*)_{ml}(Y_R)_{ij}}{2m_\Delta^2} (-\bar{\nu}_m \mathbb{P}_R \ell_i \cdot \bar{d}_i^c \mathbb{P}_R u_j^a + \bar{\ell}_m \mathbb{P}_R \ell_i \cdot \bar{u}_i^c \mathbb{P}_R u_j^a) + h.c. \right] \\
 & - \left[\frac{(Y_L^*)_{ml}(Y_L)_{ij}}{2m_\Delta^2} \bar{\nu}_m \gamma^\mu \mathbb{P}_L \ell_i \cdot \bar{d}_i^c \gamma_\mu \mathbb{P}_L u_j^a + h.c. \right] \\
 & + \frac{(Y_L^*)_{ml}(Y_L)_{ij}}{2m_\Delta^2} (\bar{\nu}_m \gamma^\mu \mathbb{P}_L \nu_i \cdot \bar{d}_i^c \gamma_\mu \mathbb{P}_L d_j^a + \bar{\ell}_m \gamma^\mu \mathbb{P}_L \ell_i \cdot \bar{u}_i^c \gamma_\mu \mathbb{P}_L u_j^a) \\
 & + \frac{(Y_R^*)_{ml}(Y_R)_{ij}}{2m_\Delta^2} \bar{\ell}_m \gamma^\mu \mathbb{P}_R \ell_i \cdot \bar{u}_i^c \gamma_\mu \mathbb{P}_R u_j^a \\
 & + \left[\frac{(Y_L^*)_{ml}(Y_R)_{ij}}{8m_\Delta^2} (\bar{\nu}_{Lm} \sigma^{\mu\nu} \mathbb{P}_R \ell_i \cdot \bar{d}_i^c \sigma_{\mu\nu} \mathbb{P}_R u_j^a - \bar{\ell}_{Lm} \sigma^{\mu\nu} \mathbb{P}_R \ell_i \cdot \bar{u}_i^c \sigma_{\mu\nu} \mathbb{P}_R u_j^a) + h.c. \right] \\
 & + \frac{(Y_S)_{ij}(Y_S^\dagger)_{lm}}{2m_S^2} [\bar{d}_m^a \gamma^\mu \mathbb{P}_R d_l^a] [\bar{d}_i^b \gamma_\mu \mathbb{P}_R d_j^b]
 \end{aligned} \tag{3.}$$

- m_ν requires nonzero Y_L .
- Minimal flavor violation and $Y_R = 0$
- from $K - \bar{K}$ mixing, $|Y_S| < 9 \times 10^{-3} \times (m_S/7\text{TeV})$.

$$|(Y_S)_{11}(Y_S)_{22}^*| < 1.92 \times 10^{-6} \times (m_S/\text{TeV})^2$$

Some considerations for $Y_R = 0$

- UV: (1) extra-dim (2) U(1) symmetry
- pheno side: If $Y_R \neq 0$, the 1-loop EDM

$$d_\ell \sim \frac{N_c}{16\pi^2} \frac{m_t}{m_\Delta^2} \text{Im}[Y_L Y_R^*]$$

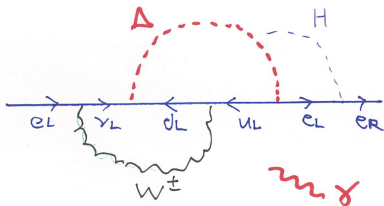
For $m_\Delta = 1\text{TeV}$, $|Y_L| \sim |Y_R| \sim 0.01$, CP phase of order one, the typical electron EDM is around 10^{-24} e-cm (currently, $|d_e| < 8.7 \times 10^{-29}$ e-cm.)

- Like in many models, a pressing theoretical issue. A plain solution: $m_\Delta \gtrsim 100\text{TeV}$ but the phenomenology..

Some considerations for $Y_R = 0$

- If $Y_R = 0$, EDM at 3-loop level.

$$d_\ell \sim \frac{\alpha N_c}{(16\pi)^3} \frac{m_\ell}{m_\Delta^2} \text{Im} \left[(Y_L)_{\ell k} V_{kj}^{CKM} (Y_L^\dagger)_{ji} U_{il}^{PMNS} \right].$$



- If $Y_L \sim 0.01$, $m_\Delta = 1\text{TeV}$, $\sim \mathcal{O}(1)$ CP phase, $|d_e| \lesssim 10^{-37}$ e-cm. Slightly larger than SM d_e .
- once d_e was measured, either the $Y_R = 0$ assumption with $m_\Delta \sim \mathcal{O}(\text{TeV})$ is out or more NP beyond the cZBM.



$$\mathcal{L} \supset \frac{1}{2} \bar{\ell}' \left(d_L^{\prime\prime} \mathbb{P}_L + d_R^{\prime\prime} \mathbb{P}_R \right) \sigma^{\mu\nu} \ell F_{\mu\nu} + h.c.$$

$$\Gamma(\ell \rightarrow \ell' \gamma) \simeq \frac{m_\ell^3}{16\pi} (|d_L^{\prime\prime}|^2 + |d_R^{\prime\prime}|^2)$$

- A straightforward calculation yields

$$d_R^{\prime\prime} = -\frac{N_c e}{16\pi^2 m_\Delta^2} \left[\left(m_{l'} (Y_R^*)_{l'q} (Y_R^T)_{ql} \right. \right. \\ \left. \left. + m_l (Y_L^*)_{l'q} (Y_L^T)_{ql} \right) \mathcal{F}_1(r_q) + m_q (Y_L^*)_{l'q} (Y_R^T)_{ql} \mathcal{F}_2(r_q) \right],$$

$q = u, c, t$, $r_q \equiv m_q^2/m_\Delta^2$. $d_L^{\prime\prime}$ by simply $Y_L \leftrightarrow Y_R$. The loop functions have limit $\mathcal{F}_1(x) \rightarrow 1/12$ and $\mathcal{F}_2(x) \rightarrow 7/6 + 2 \ln x/3$ when $x \rightarrow 0$.

- In general, $Y_R = 0$ also minimizes 1-loop cLFV.

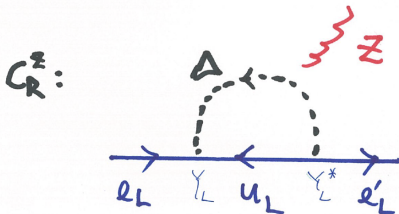
$Z \rightarrow \bar{l}l'$

- The most general $Z \rightarrow \bar{l}l'$ amplitude:

$$i\mathcal{M} = ie\bar{u}(p') \left[(c_R^Z \mathbb{P}_R + c_L^Z \mathbb{P}_L) \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{m_Z^2} \right) \gamma^\nu + \frac{1}{m_Z} \left(d_L^Z \mathbb{P}_L + d_R^Z \mathbb{P}_R \right) (i\sigma_{\mu\nu} q^\nu) \right] v(-p) \epsilon^\mu(q),$$

(c, d : dimensionless, projection on anti-particle.)

- only c_R^Z is kept in the study.



- $c_R^Z \sim Y_L^2 (M_Z/m_\Delta)^2$. $d/c_R^Z \sim (m_l/M_Z)$ and $c_L^Z/c_R^Z \sim (m_l/M_Z)^2$

- final result in numerical form:

$$\mathcal{B}(Z \rightarrow \bar{\ell}\ell') \simeq 1.46 \times 10^{-7} \left| \sum_{q=u,c,t} a_q^Z (Y_L)_{\ell'q}^* (Y_L)_{\ell q} \right|^2 \times \left(\frac{\text{TeV}}{m_\Delta} \right)^4,$$

where $a_u^Z = a_c^Z \simeq -0.125 - 0.077i = -0.1468e^{i31.63^\circ}$ and $a_t^Z = 1$.

- The imaginary part of $a_{u,c}^Z$ comes from the on-shell light quarks in the Z decay. CP violation is observable.

Numerical study

- once $\{\mu, m_S, m_\Delta, (Y_S)_{13,23,33}\}$ plus anyone of Y_L 's are fixed, all remaining 8 Y_L 's iteratively determined from m_ν matrix and U_{PMNS} . $m_\Delta = 1\text{TeV}$ and $m_S = 7\text{TeV}$. For each config. μ randomly chosen from $[0.1, 1]\text{TeV}$, $\sin^2\theta_{12,13,23}$, Dirac phase δ_{cp} : 1 sigma from global fit
- (1) All $|Y_L|$'s are less than one. (2) all TLFV (3) 1-loop cLFV :

$\mathcal{B}(\mu^+ \rightarrow e^+ \gamma)$	$< 5.7 \times 10^{-13}$, 90% C.L.
$\mathcal{B}(\tau \rightarrow \mu \gamma)$	$< 4.4 \times 10^{-8}$, 90% C.L.
$\mathcal{B}(\tau \rightarrow e \gamma)$	$< 3.3 \times 10^{-8}$, 90% C.L.
$\mathcal{B}_{\tau\mu}^Z$	$< 1.2 \times 10^{-5}$, 95% C.L.
$\mathcal{B}_{\tau e}^Z$	$< 9.8 \times 10^{-6}$, 95% C.L.
$\mathcal{B}_{\mu e}^Z$	$< 7.5 \times 10^{-7}$, 95% C.L.

4Fermi bound

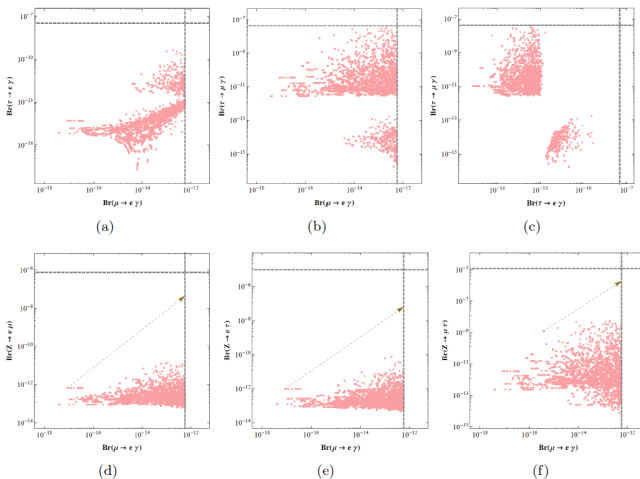
- Dimensionless parameter:

$$\epsilon_{ijkn} \equiv \frac{(Y_L)_{ik}(Y_L)_{jn}}{4\sqrt{2}G_F m_\Delta^2},$$

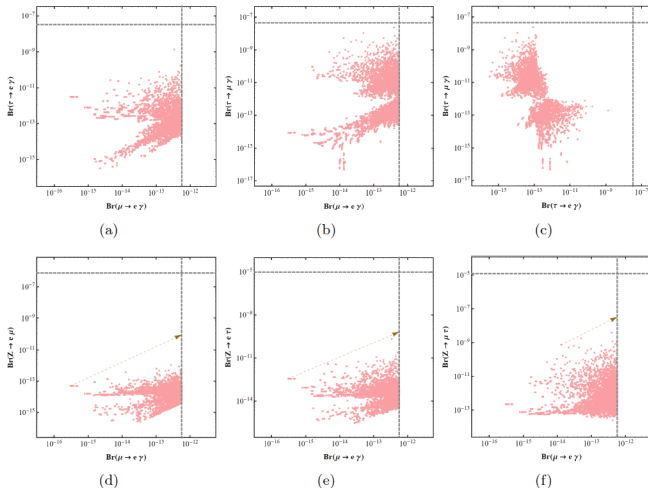
- Comprehensive study: 1008.0280

ϵ_{ee11}	10^{-3}	ϵ_{ee12}	9.4×10^{-6}	ϵ_{ee13}	3.9×10^{-3}
ϵ_{ee22}	10^{-2}	ϵ_{ee23}	10^{-3}	ϵ_{ee33}	9.2×10^{-2}
$\epsilon_{\mu\mu11}$	7.3×10^{-3}	$\epsilon_{\mu\mu12}$	9.4×10^{-6}	$\epsilon_{\mu\mu13}$	3.9×10^{-3}
$\epsilon_{\mu\mu22}$	1.2×10^{-1}	$\epsilon_{\mu\mu23}$	10^{-3}	$\epsilon_{\mu\mu33}$	6.1×10^{-2}
$\epsilon_{\tau\tau11}$	10^{-2}	$\epsilon_{\tau\tau12}$	9.4×10^{-6}	$\epsilon_{\tau\tau13}$	3.9×10^{-3}
$\epsilon_{\tau\tau22}$	1.2×10^{-1}	$\epsilon_{\tau\tau23}$	10^{-3}	$\epsilon_{\tau\tau33}$	8.6×10^{-2}
$\epsilon_{e\mu11}$	8.5×10^{-7}	$\epsilon_{e\mu12}$	9.4×10^{-6}	$\epsilon_{e\mu13}$	3.9×10^{-3}
$\epsilon_{e\mu21}$	9.4×10^{-6}	$\epsilon_{e\mu22}$	0.24	$\epsilon_{e\mu23}$	10^{-3}
$\epsilon_{e\mu31}$	3.9×10^{-3}	$\epsilon_{e\mu32}$	10^{-3}	$\epsilon_{e\mu33}$	6.6×10^{-2}
$\epsilon_{e\tau11}$	8.4×10^{-4}	$\epsilon_{e\tau12}$	9.4×10^{-6}	$\epsilon_{e\tau13}$	3.9×10^{-3}
$\epsilon_{e\tau21}$	9.4×10^{-6}	$\epsilon_{e\tau22}$	0.24	$\epsilon_{e\tau23}$	10^{-3}
$\epsilon_{e\tau31}$	3.9×10^{-3}	$\epsilon_{e\tau32}$	10^{-3}	$\epsilon_{e\tau33}$	0.2
$\epsilon_{\mu\tau11}$	9.4×10^{-4}	$\epsilon_{\mu\tau12}$	9.4×10^{-6}	$\epsilon_{\mu\tau13}$	3.9×10^{-3}
$\epsilon_{\mu\tau21}$	9.4×10^{-6}	$\epsilon_{\mu\tau22}$	0.24	$\epsilon_{\mu\tau23}$	10^{-3}
$\epsilon_{\mu\tau31}$	3.9×10^{-3}	$\epsilon_{\mu\tau32}$	10^{-3}	$\epsilon_{\mu\tau33}$	1

$m_\Delta = 1\text{TeV}$ and $|(Y_S)_{33}| = 0.0097$. Dashed lines: current limits at 90% C.L. If $Y_S \rightarrow Y_S/\lambda$, $BR \rightarrow \lambda^2 BR$.



$m_\Delta = 1\text{TeV}$ and $|(Y_S)_{33}| = 0.0097$. Dashed lines: current limits at 90%C.L. If $Y_S \rightarrow Y_S/\lambda$, $BR \rightarrow \lambda^2 BR$.





$$\mathcal{B}_{\ell\ell'}^Z \equiv \mathcal{B}(Z \rightarrow \bar{\ell}\ell') + \mathcal{B}(Z \rightarrow \ell\bar{\ell}')$$

$$\eta_{\ell\ell'} \equiv \mathcal{B}(Z \rightarrow \bar{\ell}\ell') - \mathcal{B}(Z \rightarrow \ell\bar{\ell}')$$

- Interesting at Z-factory $\sim 10^{12-13}$ per year. [NH(IH)]

	lower bounds	upper bounds (for $Y_R = 0$)
$\mathcal{B}(\mu \rightarrow e\gamma)$	3.05×10^{-16} (3.98×10^{-18})	$5.7(5.7) \times 10^{-13}$
$\mathcal{B}(\tau \rightarrow e\gamma)$	3.16×10^{-16} (2.03×10^{-18})	$2.3(0.51) \times 10^{-9}$
$\mathcal{B}(\tau \rightarrow \mu\gamma)$	4.67×10^{-17} (1.68×10^{-16})	$3.4(2.8) \times 10^{-8}$
$\mathcal{B}_{e\mu}^Z$	2.5×10^{-16} (4.9×10^{-14})	$2.2(8.7) \times 10^{-11}$
$\mathcal{B}_{e\tau}^Z$	2.9×10^{-16} (4.6×10^{-14})	$3.6(1.0) \times 10^{-10}$
$\mathcal{B}_{\mu\tau}^Z$	2.5×10^{-14} (7.8×10^{-15})	$5.5(4.5) \times 10^{-9}$
$\eta_{\mu e}$	$\begin{matrix} +.68 & (+2.1) \\ -.67 & (-.97) \end{matrix} \times 10^{-13}$	$\begin{matrix} +2.6 & (+9.3) \\ -5.4 & (-8.1) \end{matrix} \times 10^{-13}$
$\eta_{\tau e}$	$\begin{matrix} +2.4 & (+.20) \\ -.20 & (-1.2) \end{matrix} \times 10^{-12}$	$\begin{matrix} +2.3 & (+.22) \\ -.56 & (-.10) \end{matrix} \times 10^{-11}$
$\eta_{\tau\mu}$	$\begin{matrix} +2.3 & (+1.3) \\ -.78 & (-1.3) \end{matrix} \times 10^{-11}$	$\begin{matrix} +3.7 & (+3.0) \\ -8.1 & (-3.1) \end{matrix} \times 10^{-11}$

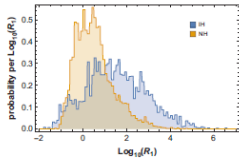
Double Ratios

- Double ratios are useful .
- Neutrino mass hierarchy can be determined if meet any of the following

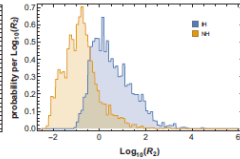
Double Ratio	IH	NH
$R_1 \equiv \mathcal{B}_{\mu\tau}^Z / \mathcal{B}(\mu \rightarrow e\gamma)$	$R_1 > 10^4$ or $R_1 < 0.1$	N.A.
$R_2 \equiv \mathcal{B}_{e\tau}^Z / \mathcal{B}(\mu \rightarrow e\gamma)$	$R_2 > 10^3$	$R_2 < 0.1$
$R_3 \equiv \mathcal{B}_{e\mu}^Z / \mathcal{B}(\mu \rightarrow e\gamma)$	$R_3 > 10^2$	$R_3 < 0.1$
$R_4 \equiv \mathcal{B}(\tau \rightarrow \mu\gamma) / \mathcal{B}(\mu \rightarrow e\gamma)$	$R_4 > 10^6$	$R_4 < 0.003$
$R_5 \equiv \mathcal{B}(\tau \rightarrow \mu\gamma) / \mathcal{B}(\tau \rightarrow e\gamma)$	N.A.	$0.03 < R_5 < 30$
$R_6 \equiv \mathcal{B}(\tau \rightarrow e\gamma) / \mathcal{B}(\mu \rightarrow e\gamma)$	$R_6 < 0.03$	N.A.
$R_7 \equiv \mathcal{B}_{\mu\tau}^Z / \mathcal{B}_{\mu e}^Z$	$R_7 < 1.0$	$R_7 > 3 \times 10^4$
$R_8 \equiv \mathcal{B}_{e\tau}^Z / \mathcal{B}_{e\mu}^Z$	N.A.	$R_8 > 10^2$
$R_9 \equiv \mathcal{B}_{\tau\mu}^Z / \mathcal{B}_{\tau e}^Z$	$R_9 < 0.01$	$R_9 > 3 \times 10^4$

- R_5 (NH) and R_7 (IH) look promising.

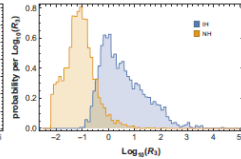
Double Ratios



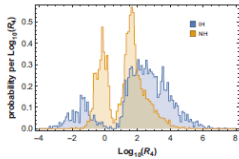
(a)



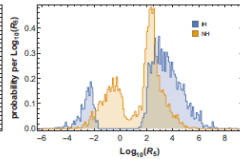
(b)



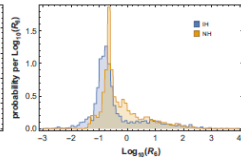
(c)



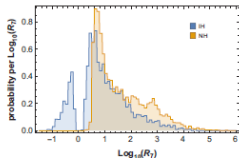
(d)



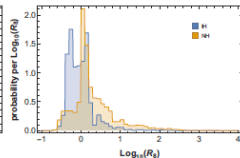
(e)



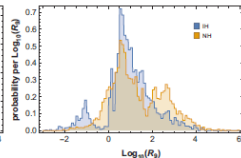
(f)



(g)



(h)



(i)

LQ decay BRs

- IH:
 - (1) $B_e^\Delta \sim 1.0$ or (2) $B_\mu^\Delta \sim 55\%$ and $B_\tau^\Delta \sim 45\%$.
- NH:

$0.7 \lesssim B_\mu^\Delta + B_\tau^\Delta \lesssim 1.0$ and $0.2 \lesssim B_\tau^\Delta \lesssim 0.8$. In other words, $B_e^\Delta \lesssim 0.3$
- Current direct search assumptions are not founded.

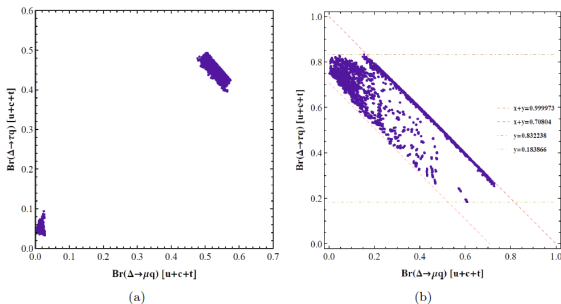


Figure 7. LQ decay branching ratios for (a) IH, (b) NH.

- Working assumption: democratic $Y_S, Y_R = 0, Y_L$ determined by m_ν
- interesting lower bounds on cLFV (Z-factory)
- double ratios and nu mass hierarchy
- definite LQ decay BRs.